Theory and Practice of Science and Technology
2025, VOL. 6, NO. 3, 23-29
DOI: 10.47297/taposatWSP2633-456904.20250603

# Optimization of Hit Probability in Deep-Water Anti-Submarine Warfare

Xuemei Yu, Hongguan Wu, Zican Dong, Jingyi Pan

Beijing Polytechnic, Beijing, 100176, China

**ABSTRACT**

This study focuses on the optimization of hit probability in deep-water anti-submarine warfare in complex sea areas, particularly considering the random errors in submarine positioning. The research is divided into three parts. First, under the condition that the submarine's depth is accurately located but its horizontal coordinates have a normal distribution error, a two-dimensional normal distribution is used to describe the uncertainty of the submarine's position. By combining three scenarios of contact fuzes and depth fuzes, the hit probability is calculated using integral methods. Through Python numerical simulations and Monte Carlo method validation, the optimal drop point (0,0) and its maximum hit probability are determined. Second, under the condition that the submarine's X, Y, and Z coordinates all have errors, and the Z coordinate follows a one-sided truncated normal distribution, a mathematical model is established. The hit probability is solved using triple integrals, and the optimal detonation depth is optimized. Finally, for the scenario where an anti-submarine aircraft carries multiple depth charges, a planar array-based bombing strategy is designed to optimize the bombing interval and maximize the hit probability. The research shows that reasonable optimization of detonation depth and bombing array can significantly improve the success rate of deep-water anti-submarine warfare. The research results have important theoretical significance and practical value for improving the combat effectiveness of anti-submarine weapons.

**KEYWORDS**

Joint probability distribution; Lagrange multiplier method; Monte Carlo method; Vectorized operation optimization

## 1 Introduction

With the increasing complexity of modern naval warfare environments, deep-water anti-submarine warfare has become a crucial component of maritime military operations. Submarines, as the main force in underwater warfare, pose significant challenges to anti-submarine operations due to their stealth and mobility. Particularly in complex sea areas, random errors in submarine positioning further increase the difficulty of anti-submarine operations. Therefore, optimizing the hit probability of deep-water anti-submarine weapons has become a key issue in improving combat effectiveness[1].

This study focuses on the optimization of hit probability in deep-water anti-submarine warfare, particularly considering the random errors in submarine positioning. The research is divided into three parts, addressing the accurate depth positioning of submarines, errors in full-direction positioning, and the optimization of multiple depth charge deployment strategies. First, under the condition that the submarine's depth is accurately located but its horizontal coordinates have a normal distribution error, a two-dimensional normal distribution is used to describe the uncertainty of the submarine's horizontal position. By combining three scenarios of contact fuzes and depth fuzes, the hit probability of depth charges at different drop points is calculated using integral methods. Through Python numerical simulations, combined with the scipy. optimize. minimize parameter optimization process and Monte Carlo method validation, the optimal drop point (0, 0) and its corresponding maximum hit probability expression are determined, verifying the effectiveness of the optimization method.

Second, under the condition that the submarine's X, Y, and Z coordinates all have errors, and the Z coordinate follows a one-sided truncated normal distribution, a corresponding mathematical model is established. By processing the joint distribution of the three coordinate axes, triple integrals are used to solve the hit probability under different scenarios of contact fuzes and depth fuzes. Numerical integration methods assist in solving the optimal detonation depth and its maximum hit probability. Finally, for the scenario where an anti-submarine aircraft carries nine depth charges, based on the above research results, the optimal detonation depth is set, and a planar array-based bombing strategy is designed. By combining the submarine's size, position distribution, and the destructive radius of the depth charges, the bombing array interval is optimized to maximize the probability of at least one depth charge hitting the submarine. Through numerical simulations and probability calculations, the optimal detonation depth and bombing array interval are determined, significantly improving the overall hit probability. The effectiveness of the bombing strategy is verified through simulations.

This paper provides a hit probability optimization solution for the practical application of deep-water anti-submarine weapons by constructing precise mathematical models and systematic numerical simulations. The research results have important theoretical guidance and practical application value for improving the combat effectiveness of deep-water anti-submarine weapons. The study shows that reasonable optimization of detonation depth and the use of scientific bombing arrays can effectively enhance the success rate of deep-water anti-submarine warfare.

## 2 Research Content

### 2.1 Optimization of Hit Probability with Accurate Depth Positioning and Horizontal Coordinate Errors

Under the condition that the submarine's depth is accurately located (150m) but its horizontal coordinates have a normal distribution error (standard deviation σ=120m), a two-dimensional normal distribution is used to describe the uncertainty of the submarine's horizontal position. By combining three scenarios of contact fuzes and depth fuzes, the hit probability of depth charges at different drop points is calculated using integral methods. Through numerical simulations and optimization methods, the optimal drop point (0, 0) is determined, and the maximum hit probability expression is derived. The results show that the hit probability is closely related to the drop point coordinates and the detonation depth of the depth fuze. The optimal solution can be precisely calculated based on the characteristics of the normal distribution and the error range.

### 2.2 Analysis of Hit Probability with Errors in Full-Direction Positioning.

Under the condition that the submarine's X, Y, and Z coordinates all have errors, with a depth positioning value of 150m, a standard deviation of 40m, and a minimum depth of 120m (following a one-sided truncated normal distribution), and the horizontal coordinate errors being the same as in Problem 1 (σ=120m), a three-dimensional joint distribution model is established. By combining different scenarios of contact fuzes and depth fuzes, the hit probability expression is solved using triple integrals. Numerical integration methods assist in optimizing the detonation depth, and the relationship between hit probability, drop point, and detonation depth is derived, providing a theoretical basis for subsequent multi-bomb deployment strategies.

### 2.3 Optimization of Multiple Depth Charge Array-Based Bombing Strategy

For the scenario where an anti-submarine aircraft carries nine depth charges, based on the parameters and optimization results from Problem 2, a planar array-based bombing strategy is designed. By combining the submarine's size, position distribution, and the destructive radius of the depth charges, the bombing array interval and the detonation depth of the depth fuze are optimized to maximize the probability of at least one depth charge hitting the submarine. Numerical simulations and probability calculations show that reasonable detonation depth and scientific bombing intervals can significantly improve the overall hit probability, providing an efficient solution for deep-water anti-submarine warfare in practical combat.

## 3 Model Assumptions and Symbol Descriptions

(1) Assume the submarine remains silent and is minimally affected by water currents.
(2) The descent speed of the depth charge is very fast, and the distance moved by the submarine is relatively small.
(3) The bombing personnel have predicted the submarine's forward direction.
(4) Assume that electronic reconnaissance aircraft continuously track the submarine and provide real-time information.

Table 1 Symbol Descriptions

| Symbol | Description |
|---|---|
| $L$ | Submarine length: 100m |
| $W$ | Submarine width: 20m |
| $H$ | Submarine height: 25m |
| $h_0$ | Submarine center depth: 150m |
| $\beta$ | Submarine heading: 90° |
| $R$ | Depth charge detonation radius: 20m |
| $Z_f$ | Detonation depth |
| $D$ | Hit range |
| $Z_b$ | Detonation depth |
| $C$ | Constant |
| $C_{L,M}$ | Product of constants |

## 4 Model Analysis and Optimization

### 4.1 Analysis of Accurate Depth Positioning with Horizontal Coordinate Errors

Under the assumption that the submarine remains silent or is minimally affected by water currents, and that the depth charge descends rapidly or the bombing personnel have predicted the submarine's forward direction and dropped bombs above it, the hit probability is maximized. Based on this assumption, this paper analyzes the vertical (y-direction) and horizontal (x-direction) aspects separately and establishes a two-dimensional joint probability distribution model. The optimal bombing strategy is solved through numerical simulations and optimization methods.

In the vertical direction, the submarine's depth is positioned at 150m, and the distance between the depth charge and the submarine affects the hit probability. Four scenarios are discussed:

Scenario 1: The distance between the depth charge and the submarine is greater than 20m, and the depth charge's destructive range cannot reach the submarine.

Scenario 2: The distance between the depth charge and the submarine is less than 20m but not in contact, and the depth charge partially affects the submarine.

Scenario 3: The depth charge contacts the submarine and triggers the contact fuze.

Scenario 4: The depth charge descends beyond the submarine's position, which is merged with Scenario 3.

The probability distribution in the vertical direction is described by a normal distribution:

$$f(y)=\frac{1}{\sqrt{2\pi\sigma_y^2}}\exp(-\frac{(y-\mu_y)^2}{2\sigma_y^2})$$

where $\mu_y$ is the mean and $\sigma_y$ is the standard deviation.

In the horizontal direction, it is assumed that electronic reconnaissance aircraft track the submarine in real-time, and four scenarios are discussed:

Scenario 1: The submarine remains silent and hidden.

Scenario 2: The submarine is unaware of tracking devices, and its speed changes have minimal impact on the depth charge's descent.

Scenario 3: The submarine's position is predicted based on water current speed and heading, and interception is performed in the forward direction (east).

Scenario 4: The submarine's speed is relatively low compared to the depth charge's descent speed, and the depth charge follows the submarine.

The probability distribution in the horizontal direction is also described by a normal distribution:

$$f(x)=\frac{1}{\sqrt{2\pi\sigma_x^2}}\exp(-\frac{(y-\mu_x)^2}{2\sigma_x^2})$$

Since the x and y directions are independent, the joint probability distribution is the product of the two normal distributions:

$$f(x,y)=\frac{1}{2\pi\sigma_x\sigma_y}\exp(-\frac{(x-\mu_x)^2}{2\sigma_x^2}-\frac{(y-\mu_y)^2}{2\sigma_y^2})$$

By plotting two-dimensional and three-dimensional contour maps using Python, it is visually evident that the hit probability is highest at the point (0,0), as shown in Figure 1.

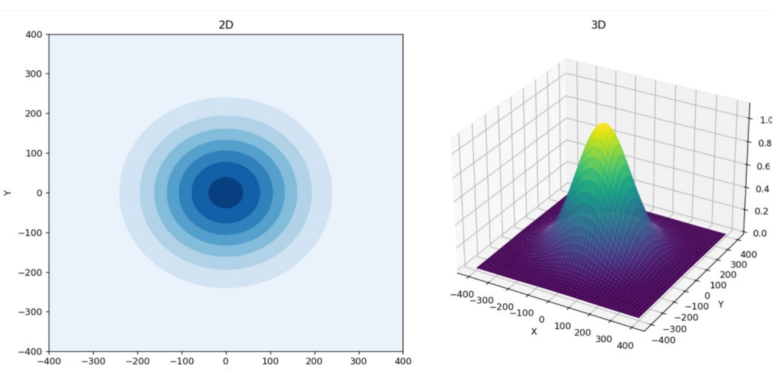


Figure 1 Two-dimensional and three-dimensional contour maps of horizontal hit probability

Assuming the submarine's center is at the origin, the horizontal coordinate error follows a normal distribution $N(0,\sigma^2)$. The probability of the drop point falling within the submarine's projection range is calculated using integrals:

$$P_X=\int_{-L/2}^{L/2}\frac{1}{\sqrt{2\pi}\,\sigma}\exp(-\frac{(x-\mu_x)^2}{2\sigma^2})dx$$

$$P_Y=\int_{-W/2}^{W/2}\frac{1}{\sqrt{2\pi}\,\sigma}\exp(-\frac{(y-\mu_y)^2}{2\sigma^2})dy$$

Using the cumulative distribution function Φ of the standard normal distribution, the expressions for

$$P_X(\mu_x)=\Phi(\frac{L/2-\mu_x}{\sigma})-\Phi(\frac{-L/2-\mu_x}{\sigma})$$

$$P_Y(\mu_y)=\Phi(\frac{W/2-\mu_y}{\sigma})-\Phi(\frac{-W/2-\mu_y}{\sigma})$$

The maximum hit probability expression is: $P_{MAX}=\max_{\mu_x,\mu_y}[P_X(\mu_x)\cdot P_Y(\mu_y)]$

During the research process, a two-dimensional joint probability distribution model was established, and numerical simulations were combined with the Lagrange multiplier method for optimization. The "scipy. optimize. minimize" function was used to optimize the drop point, ensuring the solution falls within the target area. The Monte Carlo method was employed to estimate the hit probability through random sampling, reducing computational load and improving efficiency.

## 4.2 Analysis of Hit Probability with Errors in All Directions

When there are errors in the submarine's depth positioning, the depth coordinate z follows a one-sided truncated normal distribution, and its probability density function is:

$$f_{h_0,\sigma_z,l}(v)=\frac{1}{\sigma_z}\cdot\frac{\Phi(\frac{v-h_0}{\sigma_z})}{\Phi(\frac{l-h_0}{\sigma_z})},(l<v<+\infty)$$

Where $h_0=150$ is the mean depth, $l=120$ is the minimum depth, and $\sigma_z=40$is the standard deviation. The probability distribution of the submarine's center point along the z-axis is shown in Figure 2.

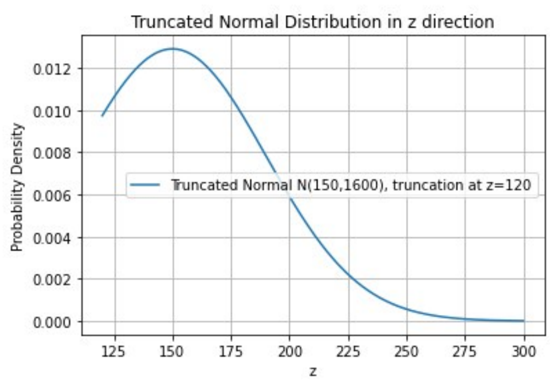


Figure 2 Probability distribution of the submarine's center point along the z-axis

Combining the normal distributions in the x and y directions, the three-dimensional joint probability distribution is: $f(x,y,v)=f(x,y)\cdot f_{h_0,\sigma_z,l}(v)$

By randomly sampling and generating a three-dimensional joint probability distribution map, the probability distribution characteristics of the submarine's position can be visually displayed.

The 3D direction and joint probability distribution slice map are shown in Figure 4.

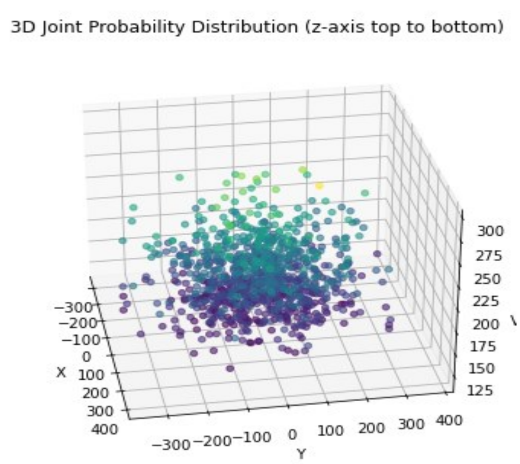


Figure 3 Three-dimensional joint probability distribution map

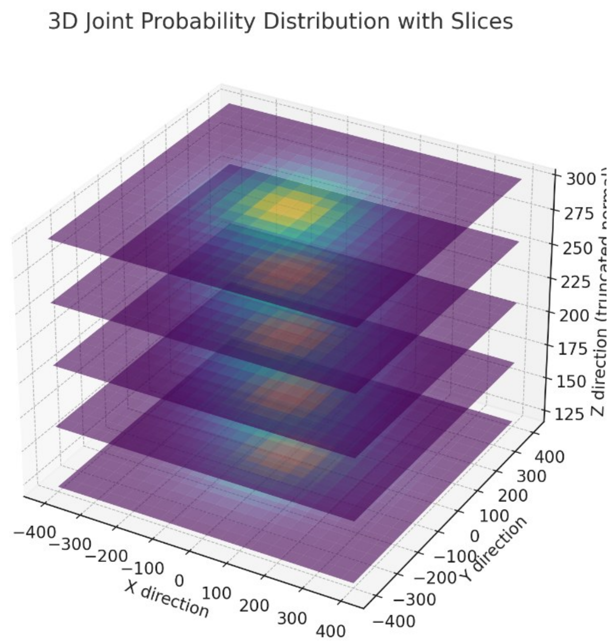


Figure 4 3D direction and joint probability distribution slice map

1 Hit Probability of Contact Fuze Detonation

The condition for contact fuze detonation is that the depth charge's drop point $(x_b, y_b, z_b)$ falls within the submarine's range, and the detonation depth $z_f$ is less than the submarine's upper surface depth: $z_f < z - \frac{H}{2}$

The hit probability

$$P_1 = \int_{z_{\min}}^{+\infty} \int_{-L/2}^{L/2} \int_{-W/2}^{W/2} f_{x,y,z}(x,y,z) \cdot 1(z_b < z - \frac{H}{2}) dxdy\, dz$$

Since the x and y directions are symmetric, the integral can be simplified to:

$$P_1 = C_{L,W} \int_{z_{\min}}^{+\infty} f_z(z) \cdot 1(z_b < z - \frac{H}{2}) dz$$

where $C_{L,W}$ is a constant representing the integral value of the x and y directions within the submarine's range.

2 Hit Probability of Depth Fuze Detonation

The condition for depth fuze detonation is that the depth charge's drop point is below the submarine and within 45 meters of the submarine's upper surface, while satisfying:

$$|x - x_b| \le \frac{L}{2} + R$$

$$|y - y_b| \le \frac{W}{2} + R$$

$$z - \frac{H}{2} < z_b < z + \frac{H}{2} + R$$

The hit probability

$$P_2 = \int_{z_{\min}}^{+\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{x,y,z}(x,y,z) \cdot 1(\sqrt{(x - x_b)^2 (y - y_b)^2 (z - z_b)^2} \le R) dxdy\, dz$$

The computational process is optimized using vectorized operations, leveraging the efficient implementation of the NumPy library to convert element-by-element processing into batch processing, significantly reducing loop iterations and improving computational efficiency. This section provides an efficient probability analysis solution for deep-water anti-submarine warfare by establishing a three-dimensional joint probability distribution model, combining different conditions of contact fuzes and depth fuzes, and optimizing the computational process through vectorized operations.

### 4.3 Multiple Depth Charge Deployment Strategy

Using Python's matplotlib, we simulated the deployment positions of nine depth charges, as shown in Figure 5. Based on the intervals in the x and y directions, a heatmap of hit probability is generated, with the depth fuze detonation depth fixed at 120m, as shown in Figure 6. Then, by analyzing the z-axis direction, the impact of the depth fuze detonation depth on hit probability is determined, as shown in Figure 7.

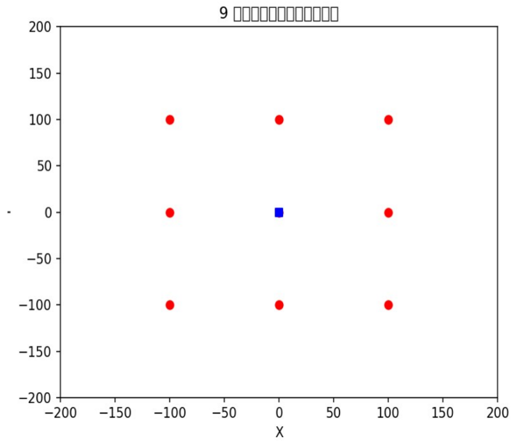


Figure 5 Simulation of nine depth charge deployment positions

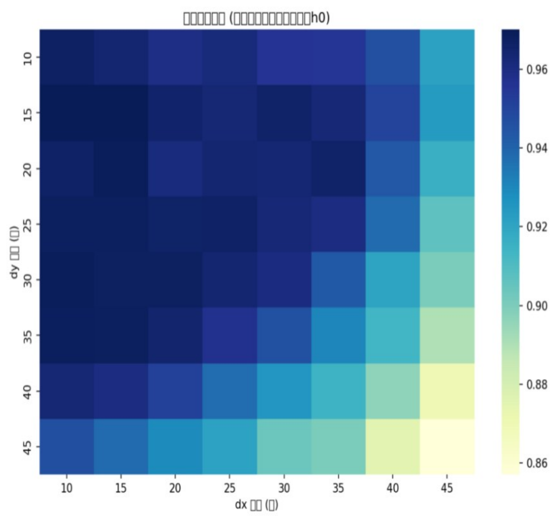


Figure 6 Heatmap of hit probability based on x and y intervals

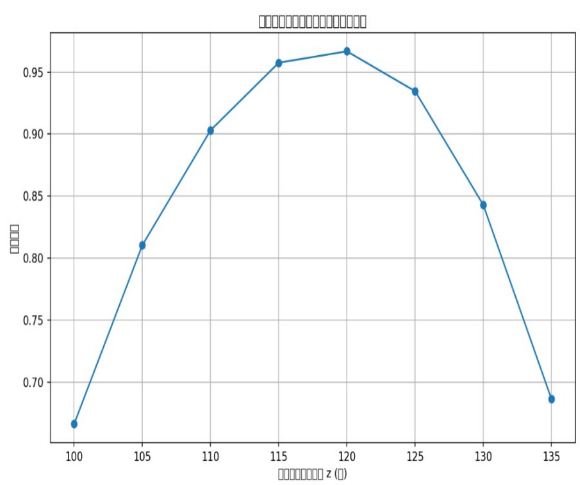


Figure 7 Impact of depth fuze detonation depth on hit probability

In deep-water anti-submarine warfare, optimizing the bombing strategy is key to improving hit probability. Based on the parameters and model analysis from Problem 2, this section designs an efficient bombing strategy by optimizing the depth fuze detonation depth and the planar interval between bombing points.

Assuming the submarine's center depth is 150m with a standard deviation of 40m, and the minimum actual depth is 120m, the depth fuze detonation depth is set to 120m to ensure the depth charge detonates near the submarine's upper surface, maximizing the hit probability. To maximize the hit probability of nine depth charges, a planar array-based bombing strategy is designed, arranging the nine depth charges in a 3x3 square array with an interval of 40m between rows and columns. This interval ensures that the depth charges cover the entire planar range of the submarine while overlapping their destructive radii to increase the probability of hitting the submarine.

The proposed bombing strategy significantly improves the hit probability of deep-water anti-submarine warfare through reasonable detonation depth and scientific bombing array design. However, further optimization is needed. In actual combat, the submarine may move, and future research could focus on dynamically adjusting the bombing interval and detonation depth. Additionally, coordinated operations involving multiple anti-submarine aircraft could be considered to further optimize the bombing strategy and improve overall hit probability.

## 5 Conclusions

This paper provides a systematic hit probability optimization solution for deep-water anti-submarine warfare through theoretical modeling and numerical simulations. Based on two-dimensional and three-dimensional joint probability distribution models, this study constructs a hit probability optimization model for deep-water anti-submarine warfare in complex sea areas, fully considering the uncertainty of submarine positioning in both horizontal (normal distribution errors) and vertical (one-sided truncated normal distribution errors) directions. By combining the Lagrange multiplier method and the Monte Carlo method, the parameters of the bombing strategy are optimized and validated. The advantages of the model are as follows:

Scientific rigor: The joint probability distribution accurately characterizes the uncertainty of submarine positioning, and the impact of hit probability is systematically analyzed by combining different scenarios of contact fuzes and depth fuzes.

Practicality: The array-based bombing strategy is designed by comprehensively considering practical factors such as submarine silence, movement characteristics, and the descent speed of depth charges, providing an operable optimization solution for actual combat.

Reliability: The Monte Carlo method is used to validate the model results, and the robustness of the solution is ensured through numerical optimization algorithms (e.g., scipy.optimize.minimize).

However, the model has the following limitations:

High computational complexity: The triple integrals of the joint probability distribution and the multi-dimensional optimization process require significant computational resources.

Limited precision of the Monte Carlo method: When the number of sampling points is insufficient, the probability estimation error may affect the reliability of the solution.

To improve the practicality and computational efficiency of the model, future work could focus on the following directions:

Upgrading optimization algorithms: Introducing parallel computing or deep learning techniques to accelerate numerical integration and optimization processes.

Enhancing the Monte Carlo method: Increasing the number of sampling points or adopting importance sampling strategies to improve the accuracy of probability estimation.

Extending to dynamic scenarios: Incorporating submarine movement prediction models to study dynamic bombing strategies for real-time battlefield environments.

## Funding

"Curvature estimation of the level set of solutions of a class of elliptic PDE"(2024X027-KXZ).

## References

[1] Jian Feng, Hao Yanbin, Li Huige. "The Close-Range Weapon of Surface Ships: Depth Charges" [J]. Global Military, 2006, (12): 48-49.

[2] Zhang Wenjun, Zhang Pingxin, Yang Liyi, et al. "Simulation of Anti-Submarine Effectiveness of Fixed-Wing Aircraft Using Self-Guided Depth Charges" [J]. Electronics Optics & Control, 2014, 21(05): 34-38+57.

[3] Liu Jianqiang. "Analysis of Position Error in Anti-Submarine Helicopter-Guided Shipborne Anti-Submarine Missile Attacks" [J]. Aero Weaponry, 2008, (06): 17-19.

[4] Jiang Xuankai, Zhao Xuetao, Jia Yue. "Analysis of Hit Probability of Self-Guided Depth Charges in Anti-Submarine Warfare" [J]. Fire Control & Command Control, 2009, 34(08): 64-67.

[5] Zhao Yapeng, Chen Yanwei, Zhang Xuyao. "Preliminary Exploration of Vertical Launch Technology for Depth Charges" [J]. Digital Ocean and Underwater Warfare, 2018, 1(01): 81-84.